The Libyan Journal of Science
University of Tripoli Vol. 29, No. 1
https://uot.ed.ly/journals/index.php/ljs

# Study of Deformation Parameters and Intrinsic Electric Quadrupole Moments for Even-Even $^{64-78}_{28}$Ni Isotopes

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**ARTICLE INFO**

***Article history:***

Received: 14/10/2025

Received in revised form 15/01/2026

Accepted: 02/02/2026

**ABSTRACT**

In this research, we have studied the reduce electric quadrupole transition probability B(E2) ↑ for the ground state $0^+ \rightarrow 2^+$ state, deformation parameter ($\beta_2$) and intrinsic electric quadrupole moments ($Q_0$) by using deformed shell model equations for even-even Nickel isotopes with neutrons number N=36, 38, 40, 42, 44, 46, 48 and 50. The results for our present calculations are compared with the experimental values. As well as, the values of B(E2) ↑, $\beta_2$ and $Q_0$ have been calculated and determined for $^{72}Ni$ isotope which have not been determined experimentally. A good agreement were obtained for all results. Also, the quantities of B(E2) ↑, $\beta_2$ and $Q_0$ for the $^{64-78}Ni$ isotopes have been plotted as a function of the number of neutrons (N) to study the effect of neutron number on them. On the other hand, it has clearly been noted from the result that $^{68}_{28}Ni$ with neutron number (40) is a stable isotope and has the spherical shape because N=40 is semi- magic number for Nickel nuclei, also it is close to the magic number (N=50).

**Keywords:** The nuclear shell model, electric transition probability (B(E2) ↑), deformation parameter ($\beta_2$), intrinsic electric quadrupole moment ($Q_0$).

## 1. Introduction

The nuclear shell model elucidates the configuration of atomic nuclei by organizing protons and neutrons into specific energy levels or "shells," similar to the atomic shell model for electrons. Central to this model is the Pauli exclusion principle, which asserts that nucleons must occupy distinct quantum states, giving rise to the notion of "magic numbers" (2, 8, 20, 28, 50, 82, 126) where closed shells lead to increased nuclear stability [1]. The shell model (SM), formulated by physicists such as Maria Goeppert-Mayer and J. Hans D. Jensen, aids in clarifying experimental findings related to nuclear stability, binding energy, and nuclear characteristics such as spins and magnetic moments [2].

In this model, each closed-shell configuration provides a useful first approximation, assuming that the system being analyzed consists of a closed-shell core along with valence particles located in a valence shell. This methodology effectively elucidates the ground state properties of nuclei [3].

The shell model has been greatly included in elucidate all properties of the nuclear levels: excitation energies, electromagnetic transition probabilities and intrinsic quadrupole moments in the medium mass nuclei, particularly study nuclei that has doubly magic nucleus, as well as study the properties of some even-even nuclei [4].

In this work, the aim is to complete past our research [5] that about Nickel nuclei has atomic number Z=28 for even-even $^{54-62}$Ni isotopes, also to calculate the deformation parameters and other values to study the shape of nuclei, as well as know the relationship between them and the magic number for Nickel isotopes that have the mass number $64 \leq A \leq 78$.

## 2. Theory

### *2.1 Deformation Parameter*

The quadrupole deformation parameter ($\beta_2$) used to describe the shape of nucleus [6], also related with the reduced electric quadrupole transition probability ($B(E2)\uparrow$) from the ground state $0^+$ to the first excited state $2^+$ that get by [7]

$$B(E2)\uparrow = 2.6\ Z^2\, E_\gamma^{-1}\, A^{-2/3} \qquad (1)$$

where:

Z: is the atomic number.

$E_\gamma$ : is the Gamma energy of transition ($0^+ \rightarrow 2^+$) in (KeV) unites.

A: is the mass number of the nucleus.

$B(E2)\uparrow$: in units of ($e^2 b^2$).

The deformation parameter ($\beta_2$) gets by the formula

$$\beta_2 = \frac{4\pi}{3Z\,R_0^2}\left[\frac{B(E2)\uparrow (e^2b^2)}{e^2}\right]^{1/2} \qquad (2)$$

where:

$R_0$ is the average nuclear radius. and $R_0^2$ is in units of barn, it given by [8]

$$R_0^2 = (1.2\,A^{1/3}\ \ \mathrm{fm})^2 \qquad (3)$$

Where:

$1\mathrm{fm} = 10^{-15}$ m, and $1\mathrm{barn} = 10^{-28}\mathrm{m}^2$.

Then we get by next

$$R_0^2 = 0.0144\,A^{2/3}\ (\mathrm{barn}) \qquad (4)$$

### *2.2 Intrinsic Quadrupole Moment*

The intrinsic quadrupole moment ($Q_0$) is defined in the intrinsic frame of reference, zero for nuclei that have a spherically symmetric charge distribution, negative for oblate nuclei and positive for prolate nuclei [9].

The (Qo) is associated with the reduced electric quadrupole transition probability B(E2) between the ground state $0^+$ and the first-excited $2^+$ [10] given by

$$Q_0 = [(16\pi B(E2)\uparrow)/(5\,e^2)]^{1/2} \qquad (5)$$

From this equation, the intrinsic quadrupole moment ($Q_0$) unit of barn (b) [11].

## 3. Results

Table 1 shown the experimental values for $B(E2)\uparrow$ and $\beta_2$ [12] for $^{64-78}$Ni isotopes. The reduced electric quadrupole transition probability $B(E2)\uparrow$ calculated by using Eq. (1) and requires values of the energy of the Gamma transitions for each Ni isotopes which are given in table 1 [12]. Values of the average radius ($R_0^2$) are calculated by Eq. (4) and quadrupole deformation parameters ($\beta_2$) by using Eq. (2), also intrinsic quadrupole moment ($Q_0$) calculating done using Eq. (5) for the isotopes with even-even $_{28}$Ni ($64 \leq A \leq 78$). The results of the quantities $B(E2)\uparrow$, $R_0^2$ , $\beta_2$ and $Q_0$ are presented respectively in table 1 for even-even nuclides $^{64-78}_{28}$Ni.

Fig. 1 shown our calculations of the reduced electric quadrupole transition probability $B(E2)\uparrow$ for $_{28}$Ni (A=64-78). The calculation of deformation parameters $\beta_2$ have been plotted as a function of neutrons number which are given in Figure 2 for $_{28}$Ni isotopes. Figure 3 showed the calculated values of intrinsic quadrupole moment ($Q_0$) as a function of (N) for even-even $_{28}$Ni isotopes.

Fig. 4 and Fig. 5 show respectively a comparison between the experimental values and calculation values of B(E2)↑ and $\beta_2$ for even-even ${}^{54-62}_{28}Ni$ [5] and present work for ${}^{64-78}_{28}Ni$ isotopes with neutrons number N=26, 28, 30, 32, 34, 36, 38, 40,42,44, 46, 48,50.

**Table 1. The transition energy ($E_\gamma$), the experimental data for ($B(E2)\uparrow$ *and* $\beta_2$) and the calculated values for ($B(E2)\uparrow, R_0^2, \beta_2$ *and* $Q_0$) in even-even Nickel isotopes (${}^{64}Ni$ to ${}^{78}Ni$) with number of neutrons (N=36-50)**

| Nuclei | N | $E_\gamma$ (KeV) | Experimental Data | | Present Work | | | |
|---|---|---|---|---|---|---|---|---|
| | | | B(E2) ↑ ($e^2b^2$) | $\beta_2$ | B(E2) ↑ ($e^2b^2$) | $R_0^2$ (b) | $\beta_2$ | $Q_0$ (b) |
| ${}^{64}Ni$ | 36 | 1345.75 | 0.0674 | 0.1688 | 0.0947 | 0.2304 | 0.1998 | 0.9757 |
| ${}^{66}Ni$ | 38 | 1424.8 | 0.06 | 0.1558 | 0.0876 | 0.2352 | 0.1883 | 0.9384 |
| ${}^{68}Ni$ | 40 | 2034.07 | 0.0261 | 0.101 | 0.0602 | 0.2399 | 0.1530 | 0.7779 |
| ${}^{70}Ni$ | 42 | 1259.6 | 0.086 | 0.179 | 0.0953 | 0.2446 | 0.1888 | 0.9788 |
| ${}^{72}Ni$ | 44 | 1096 | --- | --- | 0.1075 | 0.2492 | 0.1968 | 1.0396 |
| ${}^{74}Ni$ | 46 | 1024 | 0.064 | 0.149 | 0.1129 | 0.2538 | 0.1981 | 1.0654 |
| ${}^{76}Ni$ | 48 | 991 | --- | --- | 0.1146 | 0.2584 | 0.1960 | 1.0734 |
| ${}^{78}Ni$ | 50 | 2600 | --- | --- | 0.043 | 0.2629 | 0.1180 | 0.6575 |

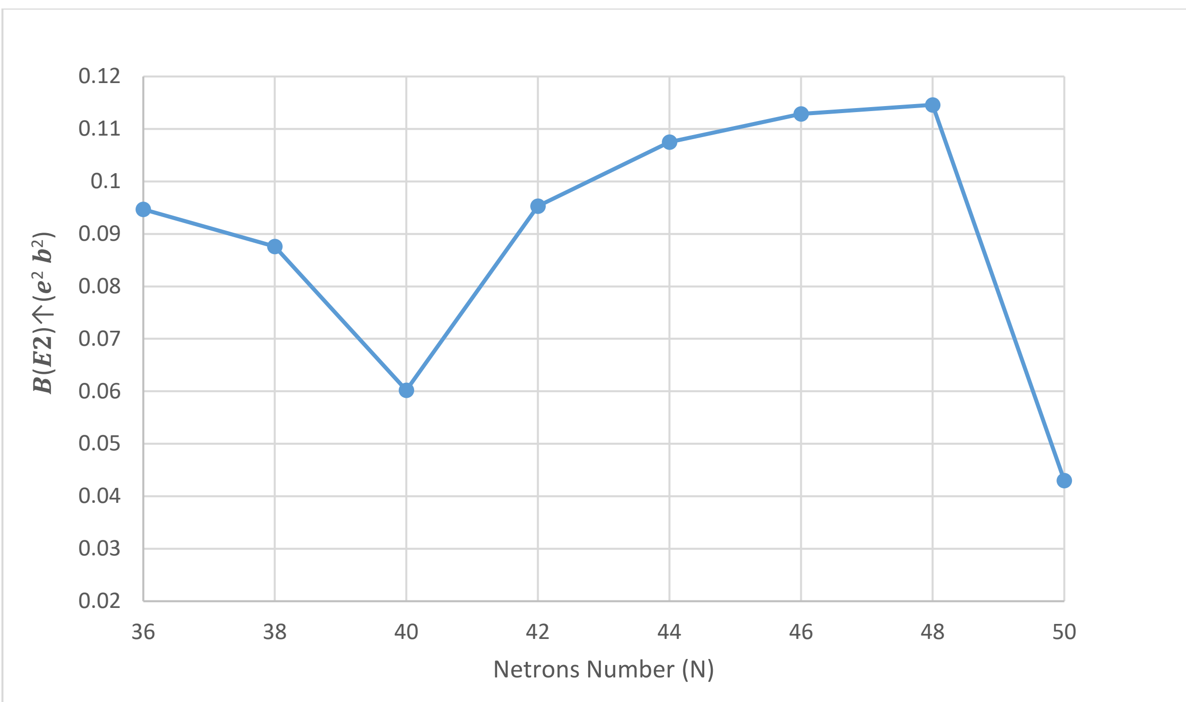


**Figure 1. The B(E2) ↑ ($e^2b^2$) calculated values as a function of (N) for ${}^{64-78}_{28}Ni$ isotopes**

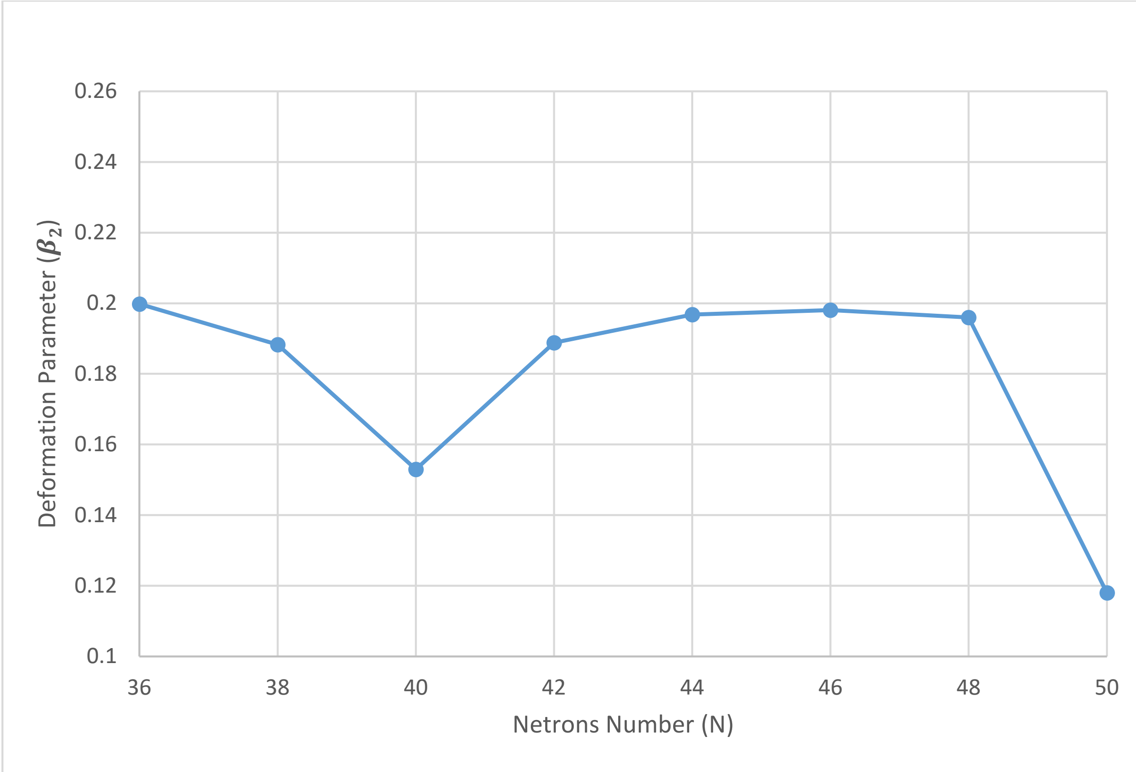


**Figure 2. The variation between the quadrupole deformation parameters $\beta_2$ and neutrons number for $^{64-78}_{28}Ni$**

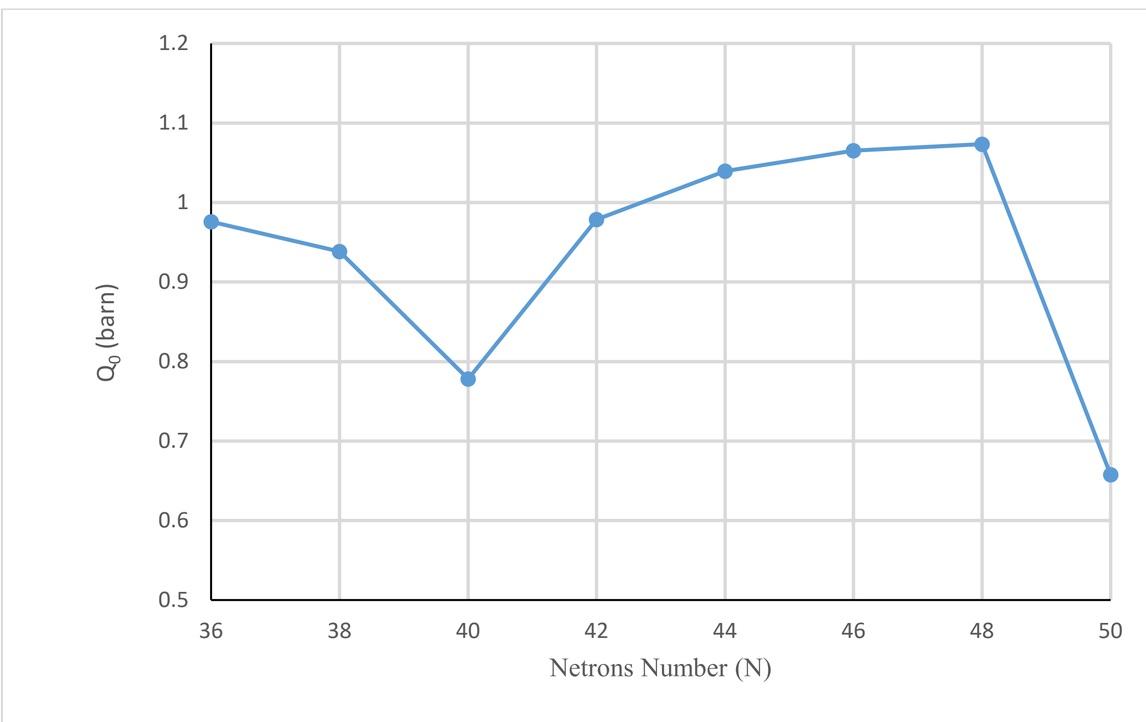


**Figure 3. Intrinsic electric quadrupole moment $Q_0$ as a function of neutrons number (N) for $^{64-78}_{28}Ni$ isotopes**

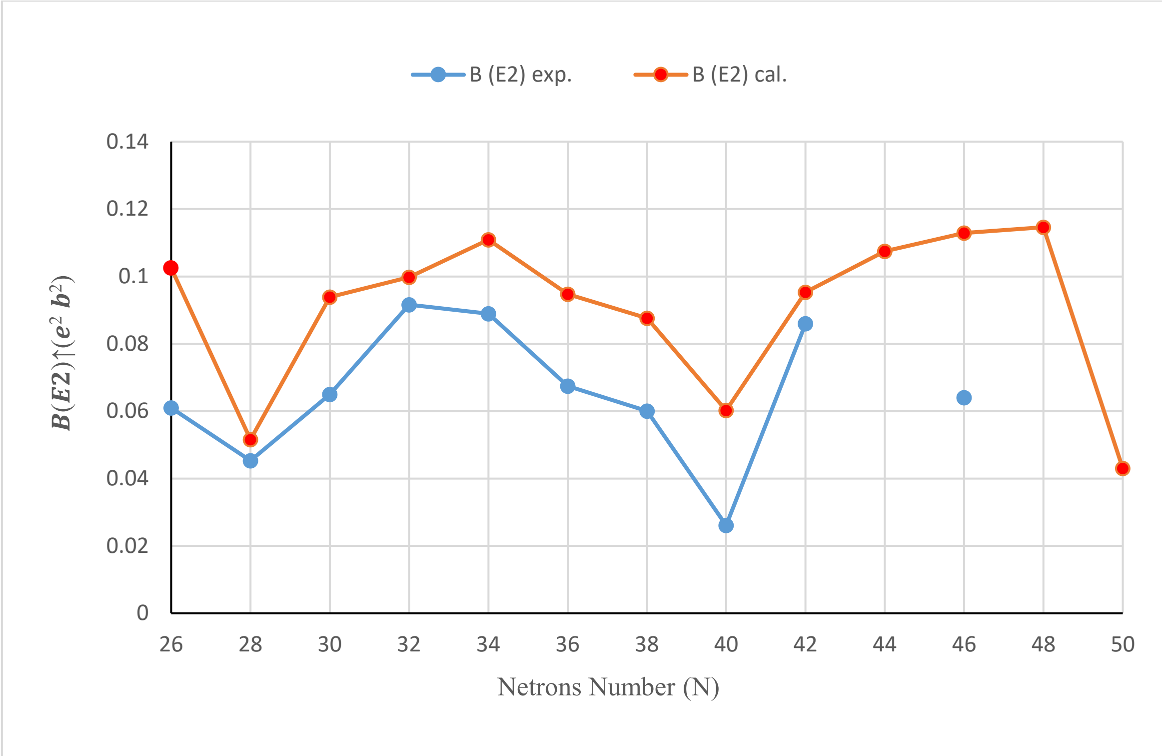


**Figure 4. Comparison between B(E2)↑$_{exp.}$ and B(E2)↑$_{cal.}$ values as a function of (N) for $^{54-78}_{28}$Ni isotopes**

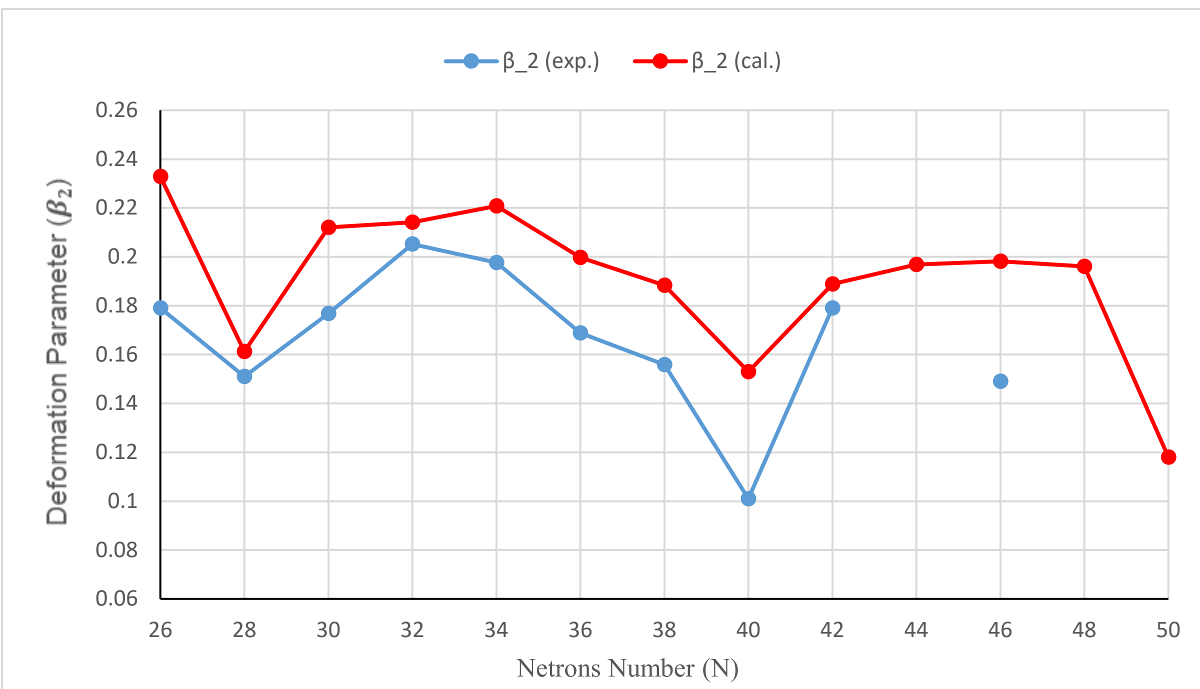


**Figure 5. Comparison between $\beta_{2\ exp.}$ and $\beta_{2\ cal.}$ values as a function of (N) for $^{54-78}_{28}$Ni isotopes**

## 4. Discussion

From Table 1 and Fig. 1, we observe that the lowest calculated value of the reduced electric quadrupole transition probability $B(E2)\uparrow$ was for the $^{78}_{28}Ni$ is equal (0.0430 $e^2b^2$) at N=50 (magic number), after that $B(E2)\uparrow$ has value 0.0602 $e^2b^2$, when neutron number (N=40), it is semi- magic number. While the largest value of $B(E2)\uparrow$ is for $^{74}_{28}Ni$ (0.1129 $e^2b^2$) at N=46 (around mid-sell).

We also seen from Table 1 and Fig. 2 that the largest value of deformation Parameter $\beta_2$ for the $^{74}_{28}Ni$ with N=46, while the lowest value is (0.1180) at N=50 for $^{78}_{28}Ni$ isotope. The highest value of intrinsic quadrupole moment ($Q_0$) (in Fig. 3) is equal to (1.0734 barn) and with N=50, the $Q_0$ has minimum value (0.6575 b).

The calculation values of the ($B(E2)\uparrow$) for $^{54-78}_{28}Ni$ isotopes were compared with the experimental values, as shown in Fig. 4. Also, the theoretical values of deformation Parameter $\beta_2$ in table 1 compared with the experimental data for $^{54-78}Ni$ isotopes, Fig. 5 shown the comparison between the experimental and calculated values of $\beta_2$ as a function of neutrons number for $^{54-78}_{28}Ni$ isotopes.

In Fig. 5, we note that deformation parameters $\beta_2$ for the calculated values are the largest of deformation at N=34 (around mid- shell between two magic number N=28 and N=40), next at the neutron number is equal 46 (midpoint between N=40 and N=50), therefore $^{62}Ni$ and $^{74}Ni$ are more reformatted, then decreasing near N=50.

From table 1, the values $B(E2)\uparrow$ and $\beta_2$ were calculated for $^{72}_{28}Ni$, $^{76}_{28}Ni$ and $^{78}_{28}Ni$ that were not available as a experimental values.

In Fig. 4 and Fig. 5, we can see that a calculated values agree with the experimental values $B(E2)\uparrow$ and $\beta_2$ respectively for even-even $^{54-78}_{28}Ni$ isotopes.

## 5. Conclusions

Quadrupole deformation parameters $\beta_2$ for Ni isotopes with a doubly- magic number (shell closed) (Z=28, N=50), have very small deformation, very close to spherical shape and more stability with high transition energy.

At N=34 and 46 for $^{62}Ni$ and $^{74}Ni$, $B(E2)\uparrow$ values are high (hence large $\beta_2$) around mid-sell. While in $^{68}Ni$ with N=40 has very small deformation (very close to spherical), so we can say that N=40 is semi-magic number for Nickel nuclei.

From the above, it can be said that the least deformation appears in the shape of the nucleus when it contains a number of neutrons or protons equal to the magic number and it increases when moving away.

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