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Simulation Approach to Model Performance Analysis Of M/D/1 Queueing Model with Encouraged Arrivals

نهج المحاكاة لتحليل أداء نموذج طابور الانتظار M/D/1 مع الوصول المشجع

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Abstract

This paper designs a comprehensive simulation framework for the M/D/1 queuing system with encouraged arrivals. A single server M/D/1 queuing model with encouraged arrivals under the steady state condition is considered. The customer in the system arrive according to a Poisson process with arrival rate of customers modified with percentage increase. This paper focuses on the distribution of the number of customers in the system in the infinite capacity M/D/1 queue. Computational Simulation is carried out to assess the performance of the proposed model. Numerical illustrations are presented to show the effect of parameters on the performance measures. R is used for analysis.

Keywords: queuing model, single server, steady state, encouraged arrivals.

الملخص

تُصمّم هذه الورقة إطار محاكاة شامل لنظام طوابير الانتظار M/D/1 مع وصولات مُشجّعة. يُؤخذ في الاعتبار نموذج طوابير انتظار M/D/1 لخادم واحد مع وصولات مُشجّعة في ظلّ حالة الاستقرار. يصل العميل في النظام وفقًا لعملية بواسون، مع تعديل معدل وصول العملاء بنسبة مئوية. تُركّز هذه الورقة على توزيع عدد العملاء في النظام في طوابير الانتظار M/D/1 ذات السعة اللانهائية. أُجريت محاكاة حاسوبية لتقييم أداء النموذج المُقترح. عُرضت رسوم توضيحية رقمية لإظهار تأثير المُعاملات على مقاييس الأداء. استُخدمت لغة البرمجة R للتحليل.

الكلمات المفتاحية: نموذج الانتظار، خادم واحد، حالة مستقرة، وصولات مشجعة.

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1. INTRODUCTION

Queuing models with encouraged arrivals provide a mathematical framework for analyzing systems where customer inflow is stimulated by external factors such as promotions, discounts, or special offers. The relevance of these models has grown due to their wide applicability in real-world scenarios, including retail, telecommunications, and service industries, where managing demand through incentives is a common strategy.

The concept extends the fundamental literature on queueing theory. Jain et al. (2014) [1] introduced the notion of reverse balking in a finite-capacity, single-server Markovian system. Subsequent work by Som and Seth (2017-2019) [2][3] analyzed the steady state of finite- capacity M/M/1/N queues with encouraged arrivals and reverse reneging. Rao et al. (2020)[4] developed a model that incorporated either encouraged or discouraged arrivals alongside a modified reneging policy. Dinushan and Walgampaya (2021)[5] compared performance measures with and without encouraged arrivals in multi-server, finite-capacity systems. Other relevant studies include [8]. For queues with deterministic service times (M/D/1), Brun and Garcia (2000)[9] derived a closedform solution for the customer distribution in finite-capacity systems, while Garcia et al. (2002) [10] provided an efficient

Computational method for the transient state. Kotb and Akhdar (2018) [11] offered an analytical solution for the steady-state M/D/1 queue with balking. Akhdar et al. (2022) [12] discussed the analytical solution for a truncated M/D/1 system with encouraged arrivals, deriving steady-state probabilities (Pn), the expected number in the system (L) , and in the queue (Lq) using an iterative method. However, their algorithm encountered significant computational instability for states $n > 4$ and arrival rates $\lambda > 0.65$, limiting its practical utility.

This study extends the work of Akhdar et al. (2022) by designing a robust simulation framework. This approach bypasses the analytical limitations, allowing for the generation of historical data under various scenarios, the reliable calculation of state probabilities for larger values of n, and a comprehensive analysis of system performance across a wider range of parameters. The primary contribution is a validated simulation tool that complements theoretical analysis for the M/D/1 queue with encouraged arrivals.

2. Model Description

2.1 *Notations and System Specification*

A single-server infinite capacity M/D/1 queuing system with encouraged arrivals is formulated under the following assumptions:

- λ: Mean arriving rate
- μ: *Mean service rate*
- n : *Number of customers in the system*
- *D: The fixed time of service between each customer and the other*
- $\rho = \lambda D$: *Utilization factor*
- L: Expected number of customers in the system
- Lq: Expected number of clientele waiting to be served
- W : Expected waiting time in the system
- Wq: q W Expected waiting time in the queue
- η: Represents the percentage increase in the number of customers computed from past or observed data
- $\lambda(1 + \eta)$: The Poisson process rate

2.2*Steady-State Probabilities*

Using the iteration method, the state probabilities are given by:

$$p_0 = e^{-\lambda(1+\eta)} p_0 + e^{-\lambda(1+\eta)} p_1 \quad (1)$$

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$$p_1 = \lambda(1+\eta)e^{-\lambda(1+\eta)}p_0 + \lambda(1+\eta)e^{-\lambda(1+\eta)}p_1 + e^{-\lambda(1+\eta)}p_2 \qquad (2)$$

$$p_n = \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^n}{n!}(p_0 + p_1) + \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^{n-1}}{(n-1)!}p_2 + \frac{e^{-\lambda(1+\eta)}[\lambda(1+\eta)]^{n-2}}{(n-2)!}p_3 + \ldots + \frac{e^{-\lambda(1+\eta)}}{0!}p_{n+1} \qquad (3)$$

For n ≥ 1, this can be rewritten as:

$$p_n = [1-\lambda(1+\eta)]\left\{e^{n\lambda(1+\eta)} + \sum_{i=1}^{n-1} e^{i\lambda(1+\eta)}(-1)^{n-i} \times \left(\frac{[i\lambda(1+\eta)]^{n-i}}{(n-i)!} + \frac{[i\lambda(1+\eta)]^{n-i-1}}{(n-i-1)!}\right)\right\}, \quad n \geq 2 \qquad (4)$$

Applying generating functions and L'Hˆopital's rule, and using the condition $\sum_{n=0}^{\alpha} P_n = 1$, the Probabilities can be expressed as:

$$p_0 = 1-\lambda(1+\eta) \qquad (5)$$

$$p_1 = [1-\lambda(1+\eta)]\left(e^{\lambda(1+\eta)} - 1\right) \qquad (6)$$

For n ≥ 2:

$$p_n = [1-\lambda(1+\eta)]\left\{e^{n\lambda(1+\eta)} + \sum_{i=1}^{n-1} e^{i\lambda(1+\eta)}(-1)^{n-i} \times \left(\frac{[i\lambda(1+\eta)]^{n-i}}{(n-i)!} + \frac{[i\lambda(1+\eta)]^{n-i-1}}{(n-i-1)!}\right)\right\} \qquad (7)$$

2.3 *Performance Measures*

The expected number of customers in the system is:

$$L = E(n) = \sum_{n=0}^{\infty} np_n \tag{8}$$

A straightforward reformulation leads to the key performance metrics:

$$L = \lambda(1+\eta) + \frac{[\lambda(1+\eta)]^2}{2[1-\lambda(1+\eta)]} \tag{9}$$

$$L_q = \frac{[\lambda(1+\eta)]^2}{2[1-\lambda(1+\eta)]}, \tag{10}$$

$$W = 1 + \frac{\lambda(1+\eta)}{2[1-\lambda(1+\eta)]} \tag{11}$$

$$W_q = \frac{\lambda(1+\eta)}{2[1-\lambda(1+\eta)]} \tag{12}$$

Note: In (11) and (12), the denominator is the base arrival rate λ, not the effective rate λ(1 + η), as these measure the time experienced by a customer whose decision to arrive was based on the original rate.

3. Simulation Study

A simulation study was conducted to assess the model's performance and overcome the computational limitations of the pure analytical approach. The goals were:

1 To compute the state probabilities Pn for a wide range of n.

2 To determine the practical limits of n (where Pn becomes negligible).

3 To calculate the performance measures L, Lq, W , and Wq.

4 To analyze the sensitivity of the system to changes in λ and η.

3.1 Simulation Design

The simulation models an infinite-capacity M/D/1 queue with encouraged arrivals. The parameters were set as follows:

Base Arrival rates: $\lambda \in \{0.01, 0.05, 0.1, 0.5, 0.9, 0.95, 0.99\}$

Service rate: $\mu = 1$ (hence $D = 1$)

Growth rates: $\eta \in \{0.01, 0.05, 0.1, 0.5, 0.9, 0.95, 0.99\}$

The system was observed for states up to $n = 50$.

3.2 Discussion of Simulation Results

The computed steady-state probabilities for selected values of λ and η are extensively tabulated in Appendix A. The key findings from the numerical analysis are summarized below:

- State Probabilities (P_n): As expected, P_n decreases as the number of customers n increases. This decay is more rapid for lower base arrival rates (λ) and higher growth rates (η). The interplay between λ and η can lead to invalid (negative) probabilities for certain parameter combinations using the analytical method, highlighting the need for a robust simulation approach. Our simulation successfully computed probabilities for $n > 4$ and $\lambda > 0.65$, overcoming the limitation in [?].

- Probability of an Empty System (P0): Figure 1 shows P_0 and P_1. For $\lambda < 0.5$, P_0 is significantly larger, and P_1 increases with η. For $\lambda > 0.5$, P_0 is much smaller, and P_1 decreases as the system is more congested. The curve for $\lambda = 0.5$ marks a clear transition point.

- Higher State Probabilities (P_n, $n \geq 2$): Figures 2-4 show that P_n becomes extremely small and effectively unobservable for low values of η and λ. For example, with $\lambda = 0.5$, $P_{\underline{n}}$ is negligible for $\eta < 0.5$ and n > 13. For high utilization ($\lambda = 0.9, 0.95, 0.99$), P_n decreases rapidly as η, λ, and n increase.

- Number of Customers: Figure 5 indicates that the number of customers is moderate for $\lambda < 0.5$ but increases dramatically for $\lambda \geq 0.5$ and $\eta > 0.5$, often exceeding 30, indicating severe congestion.

Figure 1: Computed values of the steady-state probabilities (P0, P1) for different values of η and λ.

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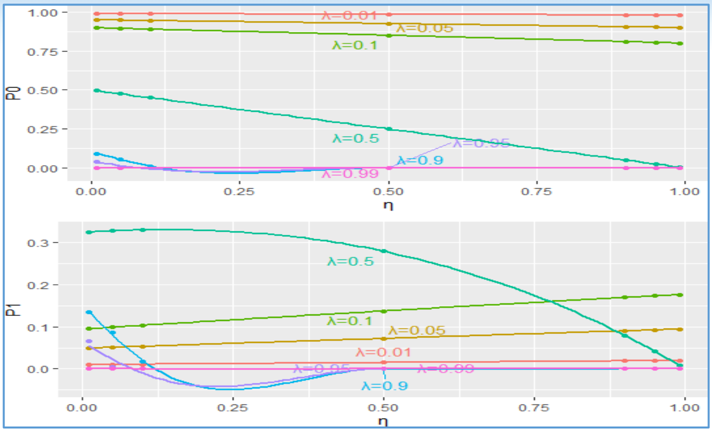

Figure 2: Steady-state probabilities Pn for λ = 0.01, 0.05, 0.1 and varying n and η. The probabilities decay rapidly and become negligible for low arrival rates.

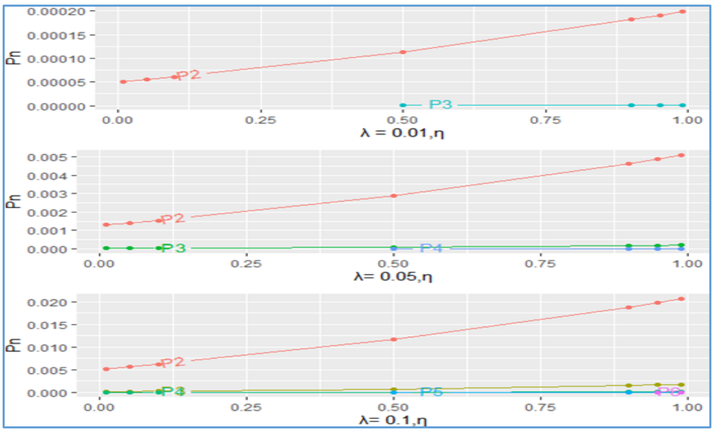

Figure 3: Steady-state probabilities Pn for λ = 0.5 and varying n and η. This rate represents a transition point in system behavior. Note that for η < 0.5 and n > 13, Pn becomes unobservable.

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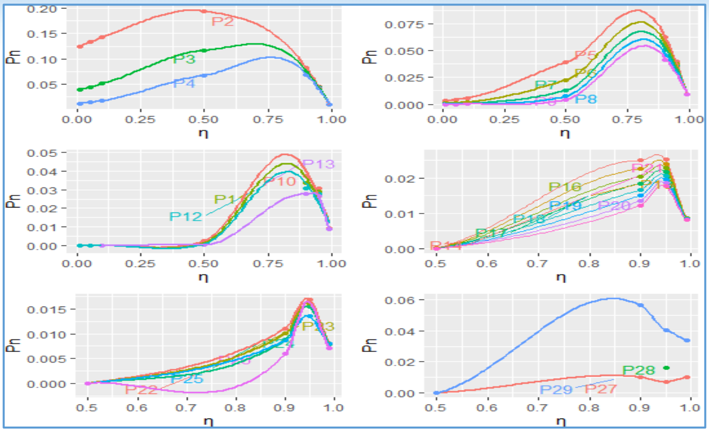

Figure 4: Steady-state probabilities Pn for λ = 0.9, 0.95, 0.99 and varying n and η. For high utilization, Pn decreases as η, λ, and n increase. Notably, for λ = 0.99, Pn is only observable for very small η (e.g., η = 0.01).

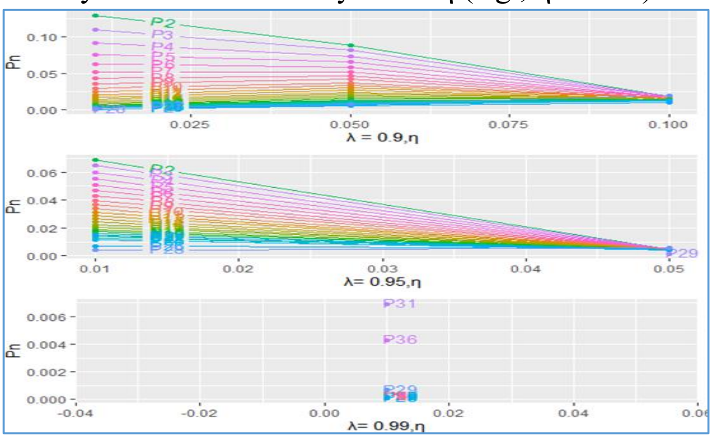

Figure 5: Computed values of the number of customers in the system (L) for different values of η and λ.

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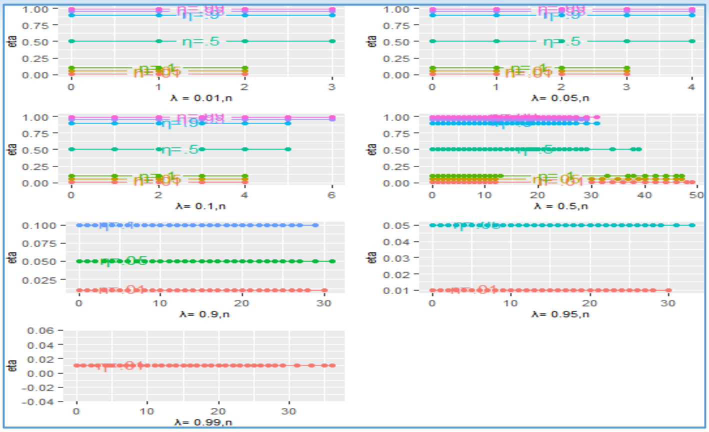

3.3 Performance Measures

The computed performance measures are summarized in Table 1. The results confirm the intuitive behavior of the system: as both the base arrival rate (λ) and the growth rate (η) increase, all four performance measures (L, Lq, W, Wq) increase. This growth becomes hyper-sensitive near full utilization ($\rho = \lambda(1 + \eta) \rightarrow 1$), where queue length and waiting times diverge to infinity, as predicted by queuing theory. For example, when (λ = 0.99, η = 0.5), the average number in the system L exceeds 100.

Table 1: Computed values of performance measures for various λ and η

λ	η	L	L_q	W	W_q
0.01	0.01	0.01015	0.00005	1.00510	0.00510
	0.05	0.05184	0.00134	1.02659	0.02659
	0.10	0.10667	0.00567	1.05617	0.05617
	0.50	0.76260	0.25760	1.51010	0.51010
	0.90	5.44900	4.54000	5.99451	4.99450
	0.95	12.32543	11.36593	12.84568	11.84568
	0.99	5000.000	4999.00	5000.500	4999.500
0.05	0.01	0.01056	0.00006	1.00531	0.00531
	0.05	0.05395	0.00145	1.02770	0.02770
	0.10	0.11116	0.00616	1.05866	0.05866
	0.50	0.81513	0.29013	1.55263	0.55263
	0.90	9.06341	8.11841	9.59091	8.59091
	0.95	199.9987	199.0012	200.500	199.500
0.10	0.01	0.01106	0.00006	1.00556	0.00556

	0.05	0.05660	0.00160	1.02910	0.02910
	0.10	0.11680	0.00680	1.06180	0.06180
	0.50	0.88611	0.33611	1.61111	0.61111
	0.90	49.9950	49.0050	50.50000	49.500
0.50	0.01	0.01511	0.00011	1.00761	0.00761
	0.05	0.07804	0.00304	1.04054	0.04054
	0.10	0.16324	0.01324	1.08824	0.08824
	0.50	1.87500	1.12500	2.50000	1.50000
0.90	0.01	0.01918	0.00018	1.00968	0.00968
	0.05	0.09999	0.00499	1.05249	0.05249
	0.10	0.21228	0.02228	1.11728	0.11728
	0.50	9.97500	9.02500	10.50000	9.50000
0.95	0.01	0.01969	0.00019	1.00994	0.00994
	0.05	0.10277	0.00527	1.05402	0.05402
	0.10	0.21862	0.02362	1.12112	0.12112
	0.50	19.98750	19.01250	20.50000	19.50000
0.99	0.01	0.02010	0.00020	1.01015	0.01015
	0.05	0.10500	0.00550	1.05525	0.05525
	0.10	0.22372	0.02472	1.12422	0.12422
	0.50	99.99750	99.00250	100.50000	99.50000

4. Findings

State Probabilities (Pn): As the number of customers n increases, Pn decreases. This decline is more pronounced when the arrival rate λ is low and the growth rate η is high. The interplay between λ and η can lead to negative probabilities for certain parameter combinations, indicating an impossible or unobserved state.

- Probability of an Empty System (P_0): Figure 1 shows P_0 and P_1. For $\lambda < 0.5$, P0 is larger and P_1 increases with η. For $\lambda > 0.5$, P0 is smaller and P_1 decreases. The curve for λ = 0.5 marks a transition point.

- Higher State Probabilities (Pn, $n \geq 2$): Figures 2-4 show that Pn becomes extremely small and effectively unobservable as η and λ decrease. For example, with $\lambda = 0.5$, Pn is unobservable for $\eta < 0.5$ and $n > 13$. For high utilization ($\lambda = 0.9, 0.95, 0.99$), Pn decreases as η, λ, and n increase.

- Number of Customers: Figure 5 indicates that the number of customers is moderate for $\lambda < 0.5$ but increases significantly for $\lambda \geq 0.5$ and $\eta > 0.5$, often exceeding 30.
- Performance Measures: Table 1 summarizes L, Lq, W, and Wq.
- The results confirm that as both the arrival rate (λ) and the growth rate (η) increase, all four performance measures increase, often dramatically near full utilization ($\rho \rightarrow 1$).

5. Conclusions

This study successfully investigated the behavior of an M/D/1 queuing system with encouraged arrivals through a robust simulation framework. The results demonstrate that:

1 The steady-state probabilities (Pn) decrease with an increasing number of customers n, especially when the base arrival rate is low and the growth rate is high.

2 The number of customers in the system becomes significantly large when the base arrival rate is high ($\lambda \geq 0.5$) and the growth rate is substantial ($\eta > 0.5$), leading to potential congestion.

3 The mean number of customers and mean waiting times (both in the system and the queue) increase as λ and η increase, with values becoming extremely large near full system utilization.

The simulation study effectively complemented the mathematical analysis by providing stable and reliable results across a wide range of parameters, including those where the pure analytical method fails. This work provides a valuable tool for analysts and engineers to model and understand the performance of systems where arrival rates can be influenced by external encouragement.

Appendix A: Steady-State Probabilities

The following table presents a sample of the calculated steady-state probabilities Pn for various values of the arrival rate λ, the growth rate η, and the number of customers n. The full dataset is available upon request.

Table 2: The computed values of the steady-state probabilities for various values ofλ, η, and n

n	$Pn, \lambda = 0.01$						
	$\eta = 0.01$	$\eta = 0.05$	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$	$\eta = 0.95$	$\eta = 0.99$
0	0.989900	0.989500	0.989000	0.985000	0.981000	0.980500	0.980100
1	0.010049	0.010444	0.010939	0.014886	0.018817	0.019307	0.019699
2	0.000051	0.000055	0.000061	0.000113	0.000182	0.000191	0.000199
3	0.000000	0.000000	0.000000	0.000001	0.000001	0.000001	0.000001
	$\lambda = 0.05$						
0	0.949500	0.947500	0.945000	0.925000	0.905000	0.902500	0.900500
1	0.049181	0.051073	0.053431	0.072043	0.090191	0.092426	0.094209
2	0.001295	0.001401	0.001538	0.002876	0.004636	0.004887	0.005091
3	0.000023	0.000026	0.000030	0.000080	0.000167	0.000182	0.000194
4	0.000000	0.000000	0.000000	0.000002	0.000005	0.000005	0.000006
$\lambda = 0.1$							
0	0.899000	0.895000	0.890000	0.850000	0.810000	0.805000	0.801000
1	0.095543	0.099086	0.103488	0.137559	0.169492	0.173325	0.176366
2	0.005248	0.005677	0.006237	0.011687	0.018855	0.019871	0.020703
3	0.000203	0.000229	0.000266	0.000715	0.001538	0.001674	0.001789
4	0.000006	0.000008	0.000009	0.000037	0.000107	0.000121	0.000133
5	0.000000	0.000000	0.000000	0.000002	0.000007	0.000008	0.000009
6				0.000000	0.000000	0.000001	0.000001
7						0.000000	0.000000
$\lambda = 0.5$							
0	0.495000	0.475000	0.450000	0.250000	0.050000	0.025000	0.005000
1	0.325208	0.327968	0.329964	0.279250	0.079285	0.041279	0.008524
2	0.124660	0.132858	0.142931	0.194235	0.082188	0.044816	0.009598
3	0.039019	0.044182	0.051154	0.116667	0.076096	0.043615	0.009716
4	0.011449	0.013817	0.017282	0.067643	0.068909	0.041567	0.009640
5	0.003315	0.004269	0.005775	0.039021	0.062245	0.039523	0.009544
6	0.000959	0.001318	0.001928	0.022506	0.056222	0.037579	0.009448
7	0.000278	0.000407	0.000644	0.012982	0.050784	0.035731	0.009354
8	0.000080	0.000126	0.000215	0.007488	0.045872	0.033974	0.009261
9	0.000023	0.000039	0.000072	0.004320	0.041435	0.032303	0.009169

10	0.000007	0.000012	0.000024	0.002492	0.0037428	0.030714	0.009077
11	0.000002	0.000004	0.000008	0.001437	0.033808	0.029204	0.008987
12	0.000001	0.000001	0.000003	0.000829	0.030538	0.027768	0.008897
13	0.000000	0.000000	0.000001	0.000478	0.027584	0.026403	0.008808
14			0.000000	0.000276	0.024916	0.025104	0.008721
15				0.000159	0.022506	0.023870	0.008634
16				0.000092	0.020329	0.022696	0.008548
$Pn, \lambda = 0.01$							
17				0.000053	0.018363	0.021580	0.008463
18				0.000031	0.016587	0.020519	0.008378
19				0.000018	0.014983	0.019510	0.008295
20				0.000010	0.013533	0.018551	0.008212
21				0.000006	0.012228	0.017636	0.008130
22				0.000003	0.011029	0.016787	0.008048
23				0.000002	0.010026	0.015882	0.007976
24				0.000001	0.008847	0.015428	0.007850
25				0.000001	0.008641	0.013485	0.007958
26				0.000000	0.005930	0.016280	0.007081
27				0.000004	0.010130	0.006944	0.009958
28				0.000008	0.000000	0.015936	0.000000
29				0.000048	0.056586	0.040564	0.033955
30				0.000000	0.000000	0.000000	0.000000
31				0.000000	0.747607		0.178457
32				0.000000	0.000000		0.000000
33				0.000000			
34				0.004448			
35				0.000000			
36				0.000000			
37				0.000000			
38				0.454102			
39				0.098633			
40				0.000000			
$\lambda = 0.9$							
0	0.091000	0.055000	0.010000				
1	0.134847	0.086505	0.016912				
2	0.129374	0.088839	0.018872				
3	0.110178	0.081429	0.018917				
4	0.091571	0.072981	0.018582				
5	0.075887	0.065243	0.018213				
6	0.062886	0.058322	0.017851				

7	0.052115	0.052138	0.017496				
8	0.043188	0.046609	0.017148				
9	0.035791	0.041667	0.016808				
10	0.029660	0.037248	0.016474				
11	0.024580	0.033299	0.016147				
12	0.020370	0.029768	0.015826				
13	0.016881	0.026611	0.015511				
14	0.013989	0.023789	0.015203				
15	0.011593	0.021267	0.014901				
16	0.009607	0.019012	0.014605				
17	0.007962	0.016996	0.014315				
18	0.006598	0.015193	0.014031				
19	0.005468	0.013582	0.013752				
20	0.004532	0.012142	0.013478				
21	0.003754	0.010854	0.013212				
22	0.003117	0.009704	0.012941				
23	0.002578	0.008674	0.012725				
24	0.002101	0.007765	0.012288				
25	0.001978	0.006992	0.012782				
26	0.000656	0.005815	0.009766				
27	0.002705	0.008039	0.019908				
28	0.000240	0.000000	0.000000				
29	0.000000	0.060559	0.117822				
30	0.092611	0.000000	0.000000				
31	0.000000	0.715806					
32		0.000000					
$\lambda = 0.95$							
0	0.040500	0.002500					
1	0.065221	0.004279					
2	0.068813	0.004840					
3	0.064937	0.004923					
4	0.059963	0.004909					
5	0.055236	0.004885					
6	0.050879	0.004860					
7	0.046868	0.004836					
8	0.043173	0.004812					
9	0.039769	0.004788					
10	0.036633	0.004764					
11	0.033745	0.004740					
12	0.031085	0.004716					

13	0.028634	0.004693					
14	0.026376	0.004669					
15	0.024297	0.004646					
16	0.022381	0.004623					
17	0.020617	0.004600					
18	0.018991	0.004577					
19	0.017494	0.004554					
20	0.016114	0.004531					
21	0.014848	0.004509					
22	0.013655	0.004485					
23	0.012677	0.004471					
24	0.011301	0.004419					
25	0.011708	0.004453					
26	0.007173	0.004473					
27	0.014984	0.003897					
28	0.004047	0.005919					
29	0.000000	0.001387					
30	0.255518	0.000000					
31	0.000000	0.075254					
32		0.000000					
33		0.146016					
34		0.000000					
$\lambda = 0.99$							
0	0.000100						
1	0.000172						
2	0.000195						
3	0.000199						
4	0.000200						
5	0.000200						
6	0.000200						
7	0.000200						
8	0.000200						
9	0.000200						
10	0.000200						
11	0.000200						
12	0.000200						
13	0.000199						
14	0.000199						
15	0.000199						
16	0.000199						

17	0.000199						
18	0.000199						
19	0.000199						
20	0.000199						
21	0.000199						
22	0.000199						
23	0.000199						
24	0.000199						
25	0.000199						
26	0.000195						
27	0.000221						
28	0.000083						
29	0.000698						
31	0.006913						
33	0.061044						
35	0.134825						
36	0.004300						

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